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C – 1362

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Sixth Semester B.Sc. Degree Examination, April 2017
First Degree Programme Under CBCSS
MATHEMATICS
Core Course – X
MM 1642 : Linear Algebra
(2013 Admn.)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first **10** questions are **compulsory**. They carry **1** mark **each**.

1. Give an example of two matrices that are equivalent but are not row equivalent.
2. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.
3. Write down all normal forms that are possible for non-zero 4×5 matrices.
4. Show by example that the union of two subspaces of a vector space need not be a subspace.
5. Prove or disprove : Subsets of linearly dependent sets are linearly dependent.
6. Find the inverse of the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.
7. If a 5×4 matrix A has a left inverse, what is the rank of A ?
8. Find the matrix of the linear mapping $f : \mathbb{R}^2 \to \mathbb{R}^3$ given by $f(x, y) = (x + 2y, 2x - y, -x)$ relative to the natural ordered bases.
9. Compute det A, where $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$.
10. Find the algebraic multiplicity of the eigen value of the matrix $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

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SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Show that there are infinitely many real 2×2 matrices A with $A^2 = -I_2$.

12. Can the vector $\begin{bmatrix} -1 \\ -4 \\ -9 \end{bmatrix}$ be written as a linear combination of the vectors, $\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 5 \\ 12 \end{bmatrix}$? Justify your answer.

13. Determine the row rank of the matrix, $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 1 & 4 \\ 0 & -1 & -2 \\ 0 & 1 & 2 \end{bmatrix}$.

14. Show that the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is row equivalent to I_2, provided $ad - bc \neq 0$.

15. Is $B = \{(x, y, z) : x + y + z = 0\}$ a subspace of $\mathbb{R}^3$? Justify your answer.

16. Show that the matrix $\begin{bmatrix} 1 & 3 & 4 & 7 \\ 2 & 3 & 5 & 8 \\ 1 & 4 & 5 & 9 \end{bmatrix}$ has neither a left inverse nor a right inverse.

17. In the vector space $\mathbb{R}^4$, let $A = \text{Span}\{(1, 2, 0, 1), (-1, 1, 1, 1)\}$ and $B = \text{Span}\{(0, 0, 1, 1), (2, 2, 2, 2)\}$. Determine $A \cap B$ and compute its dimension.

18. Describe the subspaces of $\mathbb{R}^3$.

19. Consider a skew-symmetric $n \times n$ matrix A, where n is odd. Show that A is not invertible.

20. Solve the equation, $\det \begin{bmatrix} x & a & a & a \\ a & x & a & a \\ a & a & x & a \\ a & a & a & x \end{bmatrix} = 0$.

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21. If λ is an eigen value of an invertible matrix A, prove that $\lambda \neq 0$ and that λ^{-1} is an eigen value of A^{-1}.

22. Consider the linear mapping $f : \mathbb{R}^3 \to \mathbb{R}^3$ given by $f(x, y, z) = (2x + z, y - x + z, 3z)$. Determine the matrix of f with respect to the ordered basis $\{(1, -1, 0), (1, 0, -1), (1, 0, 0)\}$.

SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry 4 marks **each**.

23. In the Cartesian place, let L be a line passing through the origin and making an angle θ with the x-axis. For every point (x, y) of the plane, let (x_L, y_L) be its reflection in the line L. Prove that $\begin{bmatrix} x_L \\ y_L \end{bmatrix} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$.

24. For matrix $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 1 & 2 & 3 & 3 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ determine the maximum number of linearly independent rows and the maximum number of linearly independent columns.

25. Find all solutions of the linear system

$x_2 + x_4 + x_5 = 1$

$x_1 + x_3 = 0$

$x_1 + x_2 + x_5 = 1$

26. Let $f : \mathbb{R}^5 \to \mathbb{R}^4$ be given by $f(a, b, c, d, e) = (a - c + 3d - e, a + 2d - e, 2a - c + 5d - e, -c + d)$. Find bases for Im f and Ker f.

27. Find the inverse of $\begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & -2 & 0 & 0 \\ 1 & 2 & 1 & -2 \\ 0 & 3 & 2 & 1 \end{bmatrix}$.

28. Find a basis for the real vector space V of complex matrices of the form $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$.

What is the dimension of V ?

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29. Show that the eigenvectors corresponding to distinct eigen values are linearly independent.
30. If A and B are invertible n × n matrices, prove that adj AB = adj B.adj A.
31. Show that similar matrices have same eigen values.

SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. a) Reduce to Hermite form, the matrix $\begin{bmatrix} 1 & -1 & 0 & -1 & -5 & -1 \\ 2 & 1 & -1 & -4 & 1 & -1 \\ 1 & 1 & 1 & -4 & -6 & 3 \\ 1 & 4 & 2 & -8 & -5 & 8 \end{bmatrix}$.

b) Show that the equations

$-5t - u + x - y = \alpha$

$t - 4u + 2x + y - z = \beta$

$-6t - 4u + x + y + z = \gamma$

$-5t - 8u + x + 4y + 2z = \delta$

have a solution if and only if $8\alpha - \beta - 11\gamma + 5\delta = 0$. Find the general solution when $\alpha = \beta = -1$, $\gamma = 3$, $\delta = 8$.

33. a) Find the rank, nullity and a basis of the kernel of the linear mapping $f : \mathbb{R}^3 \to \mathbb{R}^3$ given by $f(x, y, z) = (x + y + z, x + y + z, 2x + 2y + 2z)$.

b) Show that a linear mapping is completely and uniquely determined by its action on a basis.

34. a) Show that the matrix $\begin{bmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{bmatrix}$ is not diagonalisable.

b) Determine the transition matrix from the ordered basis $\{(1, 0, 0, 1), (0, 0, 0, 1), (1, 1, 0, 0), (0, 1, 1, 0)\}$ of $\mathbb{R}^4$ to the natural ordered basis of $\mathbb{R}^4$.

35. Given the matrix $A = \begin{bmatrix} 6\frac{1}{2} & -2\frac{1}{2} & 2\frac{1}{2} \\ -2\frac{1}{2} & 6\frac{1}{2} & -2\frac{1}{2} \\ 0 & 0 & 4 \end{bmatrix}$, find a matrix B such that $B^2 = A$.

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